

TABLE VII.  $f(k)$ , [see Eq. (31)], for various conditions.

Quantity	Direction of $k$		
	[110] 1.5	[111] 0.5	[100] 0.7
Using potassium matrix element			
(a) N and U.K. processes-arbitrary units	48	48	57
(b) $n^*$ proportional to $\tau(k)$	2.2	2.1	1.8
(c) U.K. processes only	26	34	44
(d) $n^*$ proportional to $\tau(k)$ U.K. only	3.8	2.9	2.3
Using lithium matrix element			
(a) N and U.K. processes-arbitrary units	58	62	71
(b) $n^*$ proportional to $\tau(k)$	1.7	1.6	1.4
(c) U.K. processes only	25	28	40
(d) $n^*$ proportional to $\tau(k)$ U.K. only	4.0	3.5	2.5

estimate of  $\tau(111)$ . On the other hand Eq. (31) can give a meaningful estimate of  $\tau(100)$  and  $\tau(110)$  in case of the weak  $q$  dependence.

The results show a difference of about 20% in the values of  $f(k)$  for the [110] and [100] directions counting both N and U.K. processes and using either the potassium matrix element. If only U.K. processes are counted the difference becomes nearly 50% using the potassium matrix element and 60% using the lithium matrix element.

We have also tabulated  $1-0.3V_s$  for the three principal directions. This gives the dependence of  $\tau(k)$  on sixth order cubic harmonic with  $C_1 = -0.3$ . It should be pointed out that we have no way of knowing there is no reason for  $C_0$  and  $C_1$  to be simply related the way  $A_0$  and  $A_1$  were for certain shapes of the  $E$  curves. In addition, even though the influence of  $n^*$  may be small because of the smaller size of the phonons arising from the fourth order cubic harmonic its influence on  $\tau$  is not. It is interesting to note that both matrix elements give  $\tau(110) > \tau(100)$  as would be the case if  $\tau$  were proportional to  $1-0.3V_s$ .

We conclude that if the matrix elements obtained by us are correct, then the geometry of the U.K. phonons alone is sufficient to produce appreciable anisotropy in  $\tau$  for both potassium and lithium. The velocity of sound is also highly anisotropic in the alkalis and may produce further anisotropy in  $\tau$ ; like geometrical factor  $(1/q^2)$  it is most significant in the large  $q$  region.

The highly anisotropic  $\tau(k)$  for lithium is in line with large deviation of  $n^*$  from unity for this metal noted above; however it is not clear why the same deviation does not occur in the case of potassium where the anisotropy is also large. The form of  $F(\theta)$  for lithium suggests a possible explanation for the strong temperature dependence of  $n^*$  shown in Fig. 7.  $F(\theta)$  is much

more sharply peaked in the case of lithium than in the case of potassium and although the peak is at  $\theta = 75^\circ$  scattering processes at  $\theta = 90^\circ$  are still quite heavily weighted. The wave vectors for phonons involved in scattering from  $k(110)$  at  $\theta = 90^\circ$  are quite large [approximately 50% large than for  $k(111)$  or  $k(100)$  and  $\theta = 90^\circ$ ]; as the temperature is lowered some of these phonons are no longer excited and the scattering should be changed severely. The high Debye temperature,  $\theta_D = 430^\circ K$ , suggests that there should actually be "freezing out" of phonons at nitrogen temperature even though we are interested in the Debye temperature for transverse phonons which will be lower than the specific heat  $\theta_D$ . There is, however, also the possibility that the change in  $n^*$  may be connected with the martensitic transition occurring near  $77^\circ K$ .<sup>24</sup>

The small changes in  $n^*$  with temperature for sodium and potassium may also be due to the beginning of the "freezing out" of some phonons. However, the scattering function  $F(\theta)$  is much less sharply peaked in the case of potassium than in the case of lithium and so the total scattering is much less sensitive to the freezing out of large  $q$  phonons. In addition the Debye temperatures are lower for these metals. Both factors should decrease the temperature effect in sodium and potassium.

CONCLUSIONS

The observed pressure effects in the alkalis require the assumption of an anisotropic scattering time,  $\tau(k)$ , in order to explain how relatively small increases in the warping parameters which describe the Fermi surface cause  $n^*$  to decrease. The assumption of anisotropy in  $\tau(k)$  is required both by the sign of the pressure effect, and, in the case of sodium and lithium, by its magnitude. The anisotropy in the shape of the Fermi surface is small, except possibly in the case of cesium, while the anisotropy in  $\tau$  is large. The anisotropy in  $\tau$  comes from: (1) the fact that  $1/|q|^2$  occurs as a  $k$  dependent weighting factor in the expression for  $\tau$  and (2) the fact that  $1/c_{q,s}^2$  occurs as a highly anisotropic weighting factor in the same expression.

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<sup>24</sup> C. S. Barrett, Phys. Rev. 72, 245 (1948).